Level Intended: Bachelor **Project Type:** Thesis

Collaborative Theme: IP-lookup, packet-classification, conflict detection and resolution

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Using Klee's Measure Technique to Report the Union of Overlaps of Iso-Oriented Rectangles that are Not "Tightly Covered"

Preamble

Klee's [1, 2] measure of the area of the union of n ranges on a finite plane, known simply as the 'Klee's Measure Problem', has opened up several discussions on this topic for ranges in d dimensions. To date, Overmars and Yap [3] have proven that the upper bound in calculating this measure for d greater than or equals to 2 runs in $O(n^{d/2} \log n)$ time. The idea behind is to maintain the area of a set of rectangles *implicitly* without having to represent the full boundary structure – as apposed to the *explicit* representation which would give $\Omega(n^2)$ run time in the worst case 'trellis' example. Utilizing the plane-sweep approach by Bentley and Ottmann [4], and dynamically maintaining the measure of the set of n 2D rectangles, requires only $O(\sqrt{n} \log n)$ per update.

Thesis Description

In this project, we shall attempt to adapt the main concepts and the necessary support data structures behind the Klee's measure problem to determine (and report) all overlaps from a given set R of n iso-oriented rectangles which are not tightly covered. We state an initial definition for the above as follows: The region of overlap g, which is the pairwise intersection of non-nested rectangles r_i and $r_j \in R$, is 'strictly / tightly covered' iff there exists a union of rectangles in $R - \{r_i, r_j\}$ whose measure and region equals to g.

We shall also attempt to benchmark the resulting algorithm from the above description (for 2D rectangles in R) to the runtime achieved by Eppstein and Muthukrishnan [5], and provide a visualiser to adequately view the process of detection.

References

1. V. Klee. Can the measure of $\cup[a_i, b_i]$ be computed in less than $O(n \log n)$ steps? In *The American Mathematical Monthly*, 84(4):47-57, 1977. JStor.
2. B. S. Chlebus. On Klee's measure problem in small dimensions. In B. Rovan (ed.), *SOFSEM '98: Theory and Practice of Informatics*, volume 1522, pages 304-311, 1998. LNCS, Springer-Verlag.
3. M. H. Overmars and C.-K. Yap. New upper bounds in Klee's measure problem. In *SIAM Journal of Computing*, 20(6):1034-1045, 1991. Society for Industrial and Applied Mathematics.
4. J. L. Bentley and Th. A. Ottmann. Algorithms for reporting and counting geometric intersections. In *IEEE Transactions on Computers*, 28(9):643-647, 1979. IEEE Press.
5. D. Eppstein and S. Muthukrishnan. Internet packet filter management and rectangle geometry. In *Proceedings of the Twelfth Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 827-835, 2001. ACM Press.